# The Continuity Property Via $\mathcal{I}^\omega$−OPEN SETS IN Ideal Topological Spaces

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## ABSTRACT

In this paper, we introduce introduces and investigates the notion of $\mathcal{I}^\omega$-continuous functions via class of pre$-\mathcal{I}-$open sets and we study cluster operator via this class to introduces and investigates the notion of $\mathcal{I}^\omega-$continuous functions in ideal topological spaces. The relationships between the pervious functions and other known functions are introduced and studied.

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## Keywords

Open set; Metric spaces, continuous functions.

## 1. INTRODUCTION

The continuity property is one of the fundamental concepts in point-set topology. In 1982 Hdeib [6], introduced the notion of $\omega$-open set and $\omega$-continuous function as a weaker form of open set and continuous function, respectively, in topological spaces. A subset $A$ of a space $(X, \tau)$ is called *$\omega$-open set* if for each $x \in A$, there is an open set $U_x$ containing $x$ such that $U_x - A$ is a countable set. A function $f : (X, \tau) \to (Y, \rho)$ of a topological space $(X, \tau)$ into a topological space $(Y, \rho)$ is called *$\omega$-continuous function* if for each $x \in X$ and for an open set $G$ in $Y$ containing $f(x)$, there is $\omega$-open set $U$ in $X$ containing $x$ such that $f(U) \subseteq G$. In 2009 Noiri and Noorani, [9], introduced the notion of $pre-\omega-$open set as weak form for a $\omega-$open sets. A subset $A$ of a space $X$ is called a $pre-\omega-$open set if $A \subseteq Int_\omega(Cl(A)))$, where $Int_\omega(A)$ denotes the $\omega-$interior operator of $A$ in a space $X$. A function $f : (X, \tau) \to (Y, \rho)$ is *$pre-\omega-$continuous function* if $f^{-1}(U)$ is $pre-\omega-$open set in $X$ for every open set $U$ in $Y$.

For the study of ideal topological spaces, In 2009 Ekici and Noiri, [3], introduced the notion of decompositions of continuity via $pre-\mathcal{I}-$open sets in ideal topological spaces. A subset $A$ of ideal topological space $(X, \tau, \mathcal{I})$ is called a $pre-\mathcal{I}-$open set if $A \subseteq Int(Cl^*(A))$. The complement of a $pre-\mathcal{I}-$open set is called a $pre-\mathcal{I}-$closed set. A function $f : (X, \tau, \mathcal{I}) \longrightarrow Y$ of ideal topological space $(X, \tau, \mathcal{I})$ into space $Y$ is called $pre-\mathcal{I}-$continuous function if $f^{-1}(G)$ is $pre-\mathcal{I}-$open set in $(X, \tau, \mathcal{I})$ for every open set $G$ in $Y$. In [1], we introduced the notion of $\mathcal{I}^\omega-$open set as a form stronger than $\omega-$open set and $pre-\mathcal{I}-$open set and weaker than $pre-\mathcal{I}-$open set. A subset $A$ of ideal topological space $(X, \tau, \mathcal{I})$ is called *$\mathcal{I}^\omega-$open set* if $A \subseteq Int_\omega(Cl^*(A))$. The complement of $\mathcal{I}^\omega-$open set is called *$\mathcal{I}^\omega-$closed set*. The set of all $\mathcal{I}^\omega-$open sets in $X$ denoted by $\mathcal{I}^\omega_O(X, \tau)$ and the set of all $\mathcal{I}^\omega-$closed sets in $X$ denoted by $\mathcal{I}^\omega_C(X, \tau)$., where $Int_\omega(A)$ denotes to $\omega$-interior operator of $A$ which is defined as the union of all $\omega$-open subsets of $X$ contained in $A$. $Cl_\omega(A)$ denotes to $\omega$-closure operator of $A$ which is defined as the intersection of all $\omega$-closed subsets of $X$ containing $A$.

In this paper, we introduce the continuity property via class of $\mathcal{I}^\omega-$open sets in ideal topological spaces. This paper is organized as follows. In Section 2, we introduce introduces and investigates the notion of $\mathcal{I}^\omega-$continuous functions via class of $pre-\mathcal{I}-$open sets. In Section 3, we study $\omega-$cluster operator via the class of $\mathcal{I}^\omega-$open sets to introduces and investigate the notion of $\mathcal{I}^\omega$-continuous functions in ideal topological spaces. The relationships between the pervious functions and other known functions are introduced and studied.

## 2. PRELIMINARIES

By $Cl(A)$ and $Int(A)$ we mean the closure set and the interior set of $A$ in topological space $(X, \tau)$, respectively.
An idea $\mathcal{I}$ on a topological space $(X, \tau)$ is a nonempty collection of subsets of $X$ which satisfies the following conditions:
1- if $A \in \mathcal{I}$ and $B \in A$ then $B \in \mathcal{I}$,
2- if $A \in \mathcal{I}$ and $B \in \mathcal{I}$ then $A \cup B \in \mathcal{I}$.
Applications to various fields were further investigated by Jankovic and Hamlett [2], Dontchev [5] and Arenas et al [4]. An ideal topological space is a topological space $(X, \tau)$ with an ideal $\mathcal{I}$ on $X$, and is denoted by $(X, \tau, \mathcal{I})$.

$$A^*(\mathcal{I}) = \{x \in X : U \cap A \notin \mathcal{I} \text{ for each open neighborhood } Uofx\}$$

is called the local function of asubset $A$ of $X$ with respect to $\mathcal{I}$ and $\tau$ , [7]. When there is no chance for confusion $A^*(\mathcal{I})$ is denoted by $A^*$. For every ideal topological space $(X, \tau, \mathcal{I})$, there exists a topology $\tau^*$ finer than $\tau$, generated by the base

$$\beta(\mathcal{I}, \tau) = \{U - I : U \in \tau \text{ and } I \in \mathcal{I}\}$$

Observe additionally that $Cl^*(A) = A \cup A^*$, [8], defines a Kuratowski closure operator for $\tau^*$. $Int^*(A)$ will denote the interior of $A$ in $(X, \tau^*)$.

If $(X,\tau)$ is a topological space and $\mathcal{I}$ is an ideal on $X$ then the triple $(X,\tau,\mathcal{I})$ will be called an *ideal topological space*.
The following definitions and theorem are taken from [1].

THEOREM 2.1. Let $(X,\tau,\mathcal{I})$ be an ideal topological space. If $G_\lambda$ is $\mathcal{I}^\omega$-open set for each $\lambda\in\Delta$ then $\cup_{\lambda\in\Delta}G_\lambda$ is $\mathcal{I}^\omega$-open set, where $\Delta$ is an index set.

THEOREM 2.2. Let $(X,\tau,\mathcal{I})$ be an ideal topological space. If $G$ is an open set in $(X,\tau)$ and $H$ is $\mathcal{I}^\omega$-open set then $G\cap H$ is $\mathcal{I}^\omega$-open set.

DEFINITION 2.3. Let $(X,\tau,\mathcal{I})$ be an ideal topological space and $A\subseteq X$.

(1) The $\mathcal{I}^\omega-$*closure operator* of $A$ is denoted by ${}_{\mathcal{I}^\omega}Cl(A)$ and defined by

$$ {}_{\mathcal{I}^\omega}Cl(A)=\cap\{B\subseteq X : A\subseteq B \text{ and } B\in\mathcal{I}^\omega_C(X,\tau)\}. $$

That is, ${}_{\mathcal{I}^\omega}Cl(A)$ is the intersection of all $\mathcal{I}^\omega-$closed sets containing $A$.

(2) The $\mathcal{I}^\omega-$*interior operator* of $A$ is denoted by ${}_{\mathcal{I}^\omega}Int(A)$ and defined by

$$ {}_{\mathcal{I}^\omega}Int(A)=\cup\{B\subseteq X : B\subseteq A \text{ and } B\in\mathcal{I}^\omega_O(X,\tau)\}. $$

That is, ${}_{\mathcal{I}^\omega}Int(A)$ is the union of all $\mathcal{I}^\omega-$open sets contained in $A$.

THEOREM 2.4. A subset $A$ of an ideal topological space $(X,\tau,\mathcal{I})$ is $\mathcal{I}^\omega-$closed set if and only if $CL_\omega(Int^*(A))\subseteq A$.

## 3. $\mathcal{I}^\omega$-CONTINUOUS FUNCTIONS

DEFINITION 3.1. Let $(X,\tau,\mathcal{I})$ be an ideal topological space and $(Y,\rho)$ be topological space Then the S-map $f:(X,\tau,\mathcal{I})\to(Y,\rho)$ is called $\mathcal{I}^\omega$*-continuous function* if $f^{-1}(V)$ is $\mathcal{I}^\omega$-open set in $(X,\tau,\mathcal{I})$ for every open set $V$ in $Y$.

It is clear that every $\omega$-continuous function is $\mathcal{I}^\omega$-continuous function but the converse of this fact no need to be true.

EXAMPLE 3.2. Let $f:(\mathbb{R},\tau,\mathcal{I})\to(Y,\rho)$ be a function defined by

$$ f(x)=\begin{cases} a, & x\in\mathbb{R}-\{2\}\\ b, & x=2\end{cases} $$

where $Y=\{a,b\}$,

$$ \tau=\{\emptyset,\mathbb{R}\},\ \mathcal{I}=\{\emptyset,\{1\}\},\ \text{and } \rho=\{\emptyset,Y,\{b\}\}. $$

The function $f$ is $\mathcal{I}^\omega$-continuous, since $f^{-1}(\{b\})=\{2\}$ and $f^{-1}(Y)=\mathbb{R}$ are $\mathcal{I}^\omega$-open sets in $(\mathbb{R},\tau,\mathcal{I})$. The function $f$ is not $\omega$-continuous, since $f^{-1}(\{b\})=\{2\}$ is not $\omega$-open set.

It is clear that every pre$-\mathcal{I}-$continuous function is $\mathcal{I}^\omega-$continuous function but the converse of this fact no need to be true.

EXAMPLE 3.3. Let $f:(\mathbb{R},\tau,\mathcal{I})\to(Y,\rho)$ be a function defined by $f(2)=a$ and $f(1)=f(3)=b$, where $Y=\{a,b\}$

$$ \tau=\{\emptyset,\mathbb{R}\},\ \mathcal{I}=\{\emptyset,\{1\}\},\ \text{and } \rho=\{\emptyset,Y,\{a\}\}. $$

The function $f$ is $\mathcal{I}^\omega$-continuous, since $f^{-1}(\{b\})=\{1,3\}$ and $f^{-1}(Y)=\mathbb{R}$ are $\mathcal{I}^\omega$-open sets in $(\mathbb{R},\tau,\mathcal{I})$. The function $f$ is not $pre-\mathcal{I}$-continuous, since $f^{-1}(\{b\})=\{1,3\}$ is not $pre-\mathcal{I}$-open set.

It is clear that every $\mathcal{I}^\omega-$continuous function is $pre-\omega-$continuous function but the converse of this fact no need to be true.

EXAMPLE 3.4. Let $f:(\mathbb{R},\tau,\mathcal{I})\to(Y,\rho)$ be a function defined by $f(2)=a$ and $f(1)=b$, where $Y=\{a,b\}$

$$ \tau=\{\emptyset,\mathbb{R}\},\ \mathcal{I}=\{\emptyset,\{1\}\},\ \text{and } \rho=\{\emptyset,Y,\{a\}\}. $$

The function $f$ is $pre-\omega-$continuous, since $f^{-1}(\{b\})=\{1\}$ and $f^{-1}(Y)=\mathbb{R}$ are $pre-\omega-$open sets in $(\mathbb{R},\tau,\mathcal{I})$. The function $f$ is not $\mathcal{I}^\omega-$continuous, since $f^{-1}(\{b\})=\{1\}$ is not $\mathcal{I}^\omega-$open set.

THEOREM 3.5. A function $f:(X,\tau,\mathcal{I})\to(Y,\rho)$ of an ideal topological space $(X,\tau,\mathcal{I})$ into a space $(Y,\rho)$ is $\mathcal{I}^\omega$-continuous if and only if $f^{-1}(F)$ is $\mathcal{I}^\omega$-closed set in $(X,\tau,\mathcal{I})$ for every closed set $F$ in $Y$.

THEOREM 3.6. If $f:(X,\tau,\mathcal{I})\to(Y,\rho)$ is $\mathcal{I}^\omega$-continuous function if and only if for each $x\in X$ and each open set $U$ in $Y$ with $f(x)\in U$, there exists $\mathcal{I}^\omega$-open set $V$ in $(X,\tau,\mathcal{I})$ such that $x\in V$ and $f(V)\subseteq U$.

PROOF. Suppose that $f$ is $\mathcal{I}^\omega$-continuous function. Let $x\in X$ and $U$ be any open set in $Y$ containing $f(x)$. Put $V=f^{-1}(U)$. Since $f$ is a $\mathcal{I}^\omega$-continuous then $V$ is $\mathcal{I}^\omega$-open set in $(X,\tau,\mathcal{I})$ such that $x\in V$ and $f(V)\subseteq U$.
Conversely, Let $U$ be any open set in $Y$. For each $x\in f^{-1}(U)$, $f(x)\in U$. Then by the hypothesis, there exists $\mathcal{I}^\omega$-open set $V_x$ in $(X,\tau,\mathcal{I})$ such that $x\in V_x$ and $f(V_x)\subseteq U$. This implies, $V_x\subseteq f^{-1}(U)$ and so $f^{-1}(U)=\cup_{x\in f^{-1}(U)}V_x$. Hence by Theorem (2.1),

$$ f^{-1}(U)=\cup_{x\in f^{-1}(U)}V_x $$

is $\mathcal{I}^\omega$-open set in $(X,\tau,\mathcal{I})$. That is, $f$ is $\mathcal{I}^\omega$-continuous. □

THEOREM 3.7. A function $f:(X,\tau,\mathcal{I})\to(Y,\rho)$ is $\mathcal{I}^\omega$-continuous of ideal topological space $(X,\tau,\mathcal{I})$ into a space $(Y,\rho)$ if and only if

$$ f[{}_{\mathcal{I}^\omega}Cl(A)]\subseteq {}_\rho Cl(f(A)) \text{ for all } A\subseteq X. $$

PROOF. Let $f$ be $\mathcal{I}^\omega$-continuous function and $A$ be any subset of $X$. Then ${}_\rho Cl(f(A))$ is a closed set in $Y$. Since $f$ is $\mathcal{I}^\omega$-continuous then by Theorem (3.5), $f^{-1}[{}_\rho Cl(f(A))]$ is $\mathcal{I}^\omega$-closed set in $(X,\tau,\mathcal{I})$. That is,

$$ {}_{\mathcal{I}^\omega}Cl\big[f^{-1}[{}_\rho Cl(f(A))]\big]=f^{-1}[{}_\rho Cl(f(A))]. $$

Since $f(A)\subseteq {}_\rho Cl(f(A))$ then $A\subseteq f^{-1}[{}_\rho Cl(f(A))]$. This implies,

$$ {}_{\mathcal{I}^\omega}Cl(A)\subseteq {}_{\mathcal{I}^\omega}Cl\big[f^{-1}[{}_\rho Cl(f(A))]\big]=f^{-1}[{}_\rho Cl(f(A))]. $$

Hence $f[{}_{\mathcal{I}^\omega}Cl(A)]\subseteq {}_\rho Cl(f(A))$.
Conversely. let $H$ be any closed set in $Y$, that is, ${}_\rho Cl(H)=H$. Since $f^{-1}(H)\subseteq X$. Then by the hypothesis,

$$ f\big[{}_{\mathcal{I}^\omega}Cl[f^{-1}(H)]\big]\subseteq {}_\rho Cl[f(f^{-1}(H))]\subseteq {}_\rho Cl(H)=H. $$

This implies, ${}_{\mathcal{I}^\omega}Cl[f^{-1}(H)]\subseteq f^{-1}(H)$. Hence ${}_{\mathcal{I}^\omega}Cl[f^{-1}(H)]=f^{-1}(H)$, that is, $f^{-1}(H)$ is $\mathcal{I}^\omega$-closed set in $(X,\tau,\mathcal{I})$. Hence by Theorem (3.5), $f$ is $\mathcal{I}^\omega$-continuous. □

THEOREM 3.8. A function $f:(X,\tau,\mathcal{I})\to(Y,\rho)$ is $\mathcal{I}^\omega$-continuous of ideal topological space $(X,\tau,\mathcal{I})$ into a space $(Y,\rho)$ if and only if

$$ {}_{\mathcal{I}^\omega}Cl(f^{-1}(B))\subseteq f^{-1}({}_\rho Cl(B)) \text{ for all } B\subseteq Y. $$

PROOF. Let $f$ be $\mathcal{I}^\omega$-continuous function and $B$ be any subset of $Y$. Then ${}_\rho Cl(B)$ is a closed set in $Y$. Since $f$ is $\mathcal{I}^\omega$-continuous then by Theorem (3.5), $f^{-1}[{}_\rho Cl(B)]$ is $\mathcal{I}^\omega$-closed set in $(X,\tau,\mathcal{I})$. That is,

$$_{\mathcal{I}^\omega}Cl\big[f^{-1}[{}_\rho Cl(B)]\big] = f^{-1}[{}_\rho Cl(B)].$$

Since $B \subseteq {}_\rho Cl(B)$ then $f^{-1}(B) \subseteq f^{-1}[{}_\rho Cl(B)]$. This implies,

$$_{\mathcal{I}^\omega}Cl(f^{-1}(B)) \subseteq {}_{\mathcal{I}^\omega}Cl\big[f^{-1}[{}_\rho Cl(B)]\big] = f^{-1}[{}_\rho Cl(B)].$$

Hence ${}_{\mathcal{I}^\omega}Cl(f^{-1}(B)) \subseteq f^{-1}[{}_\rho Cl(B)]$.
Conversely, let $H$ be any closed set in $Y$, that is, ${}_\rho Cl(H) = H$. Since $H \subseteq Y$. Then by the hypothesis,

$$_{\mathcal{I}^\omega}Cl(f^{-1}(H)) \subseteq f^{-1}({}_\rho Cl(H)) = f^{-1}(H).$$

This implies, ${}_{\mathcal{I}^\omega}Cl[f^{-1}(H)] \subseteq f^{-1}(H)$. Hence ${}_{\mathcal{I}^\omega}Cl[f^{-1}(H)] = f^{-1}(H)$, that is, $f^{-1}(H)$ is $\mathcal{I}^\omega$-closed set in $(X,\tau,\mathcal{I})$. Hence by Theorem (3.5), $f^{-1}(H)$ is $\mathcal{I}^\omega$-closed set in $(X,\tau,\mathcal{I})$. That is, $f$ is $\mathcal{I}^\omega$-continuous. □

THEOREM 3.9. A function $f : (X,\tau,\mathcal{I}) \to (Y,\rho)$ is $\mathcal{I}^\omega$-continuous of ideal topological space $(X,\tau,\mathcal{I})$ into a space $(Y,\rho)$ if and only if

$$f^{-1}({}_\rho Int(B)) \subseteq {}_{\mathcal{I}^\omega}Int[f^{-1}(B)] \text{ for all } B \subseteq Y.$$

PROOF. Let $f$ be $\mathcal{I}^\omega$-continuous function and $B$ be any subset of $Y$. Then ${}_\rho Int(B)$ is an open set in $Y$. Since $f$ is $\mathcal{I}^\omega$-continuous then $f^{-1}[{}_\rho Int(B)]$ is $\mathcal{I}^\omega$-open set in $(X,\tau,\mathcal{I})$. That is,

$$_{\mathcal{I}^\omega}Int\big[f^{-1}[{}_\rho Int(B)]\big] = f^{-1}[{}_\rho Int(B)].$$

Since ${}_\rho Int(B) \subseteq B$ then $f^{-1}[{}_\rho Int(B)] \subseteq f^{-1}(B)$. This implies,

$$f^{-1}[{}_\rho Int(B)] = {}_{\mathcal{I}^\omega}Int\big[f^{-1}[{}_\rho Int(B)]\big] \subseteq {}_{\mathcal{I}^\omega}Int(f^{-1}(B)).$$

Hence $f^{-1}({}_\rho Int(B)) \subseteq {}_{\mathcal{I}^\omega}Int[f^{-1}(B)]$.
Conversely, let $U$ be any open set in $Y$, that is, ${}_\rho Int(U) = U$. Since $U \subseteq Y$. Then by the hypothesis,

$$f^{-1}(U) = f^{-1}({}_\rho Int(U)) \subseteq {}_{\mathcal{I}^\omega}Int[f^{-1}(U)].$$

This implies, $f^{-1}(U) \subseteq {}_{\mathcal{I}^\omega}Int[f^{-1}(U)]$. Hence $f^{-1}(U) = {}_{\mathcal{I}^\omega}Int[f^{-1}(U)]$, that is, $f^{-1}(U)$ is $\mathcal{I}^\omega$-open set in $(X,\tau,\mathcal{I})$. Hence $f$ is $\mathcal{I}^\omega$-continuous. □

DEFINITION 3.10. A function $f : (X,\tau,\mathcal{I}) \to (Y,\rho)$ of a ideal topological space $(X,\tau,\mathcal{I})$ into a space $(Y,\rho)$ is called a $\mathcal{I}^\omega$*−closed function* if $f(G)$ is a closed set in $(Y,\rho)$ for every $\mathcal{I}^\omega-$closed set $G$ in $(X,\tau,\mathcal{I})$.

THEOREM 3.11. Let $f : (X,\tau,\mathcal{I}) \to (Y,\rho)$ and $h : (Y,\rho) \to (Z,\gamma)$ be two functions. Then $h \circ f$ is $\mathcal{I}^\omega-$closed function if $h$ is a closed function and $f$ is $\mathcal{I}^\omega-$closed function

PROOF. Let $U$ be $\mathcal{I}^\omega-$closed set in $(X,\tau,\mathcal{I})$. Since $f$ is $\mathcal{I}^\omega-$closed function then $f(U)$ is a closed set in $Y$. Since $h$ is closed function then $h[f(U)] = (h \circ f)(U)$ That is, $h \circ f$ is a $\mathcal{I}^\omega-$closed function. □

THEOREM 3.12. A function $f : (X,\tau,\mathcal{I}) \to (Y,\rho)$ is a $\mathcal{I}^\omega-$closed function if and only if ${}_\rho Cl[f(A)] \subseteq f[{}_{\mathcal{I}^\omega}Cl(A)]$ for all $A \subseteq X$.

PROOF. Suppose that $f$ is $\mathcal{I}^\omega$-closed function and $A$ be any subset of $X$. Since ${}_{\mathcal{I}^\omega}Cl(A)$ is $\mathcal{I}^\omega-$closed set in $(X,\tau,\mathcal{I})$ and $f$ is $\mathcal{I}^\omega-$closed function then $f[{}_{\mathcal{I}^\omega}Cl(A)]$ is a closed set in $Y$. That is,

$$_\rho Cl\big[f[{}_{\mathcal{I}^\omega}Cl(A)]\big] = f[{}_{\mathcal{I}^\omega}Cl(A)].$$

Since $A \subseteq {}_{\mathcal{I}^\omega}Cl(A)$ then $f(A) \subseteq f[{}_{\mathcal{I}^\omega}Cl(A)]$. This implies,

$$_\rho Cl[f(A))] \subseteq {}_\rho Cl\big[f[{}_{\mathcal{I}^\omega}Cl(A)]\big] = f[{}_{\mathcal{I}^\omega}Cl(A)].$$

Hence ${}_\rho Cl[f(A)] \subseteq f[{}_{\mathcal{I}^\omega}Cl(A)]$.
Conversely, let $F$ be any $\mathcal{I}^\omega$-closed set in $(X,\tau,\mathcal{I})$, that is, ${}_{\mathcal{I}^\omega}Cl(F) = F$. Since $F \subseteq X$. Then by the hypothesis,

$$_\rho Cl[f(F)] \subseteq f[{}_\rho Cl(F)] = f(F).$$

This implies, ${}_\rho Cl[f(F)] \subseteq f(F)$. Hence ${}_\rho Cl[f(F)] = f(F)$, that is, $f(F)$ is a closed set in $Y$. Hence $f$ is $\mathcal{I}^\omega$-closed function. □

## 4. REFERENCES


[1] A. Saif and K. A. Alasly (2021), On $\mathcal{I}^\omega$-open sets in ideal topological spaces, (Submitted).

[2] D. Jankovic, and T. Hamlett, New topologies from old vian ideals, Amer. Math. Monthly, 97(4) (1990), 295-310.

[3] E. Ekici and T. Noiri, On subsets and decompositions of continuity in ideal topological spaces, Arab. J. Sci. Eng. Sect. A Sci. 34(2009), 165-177.

[4] F. Arenas, J. Dontchev and M. Puertas, Idealization of some weak separation axioms, Acta Math. Hungar., 89(1) (2000), 47-53.

[5] J. Dontchev, Strong B-sets and another decomposition of continuity, Acta Math. Hungar., 75(3) (1997), 259-265.

[6] H. Z. Hdeib, $w-$closed mappings, Revista Colombiana de Matematicas, 16 (1982), 65-78.

[7] K. Kuratowski, Topology, Vol. 1, Academic Press, New York, (1966).

[8] R. Vaidyanathaswamy, The localization theory in set topology, Proc. Indian Acad. Sci., Sect A, 20(1944), 51-61.

[9] T. Noiri, A. Al-omari and M. Noorani, Weak forms of $\omega-$open sets and decompositions of continuity, European Journal of Pure and Applied Mathematics 1, (2009), 73-84.